DVA D05 Queuing Theory Exercises

February 10, 2008

Exercise 1

Assume that there are four employees at the telephone support in a software company. They work between 8am and 5pm, answering questions from the clients. If all employees are busy when a client calls, the client can wait in an unlimited queue. The company has 1000 clients that each calls in average once every 5th day. On average, how long time can an employee spend on a client's call for the system to be stable?

$U = \frac{\lambda s}{c}$ set $U = 1$ and solve for s, (using minutes as unit): $s = \frac{c}{\lambda} = \frac{4}{(1000/(5*9*60))} = 10,8$minutes.

Exercise 2

Consider a town with 50.000 inhabitants. Each person in the town calls someone else in the town on average 3 times each day (24 hours). Each call lasts in average 6 minutes. We can assume that no calls are blocked. How many persons are in average talking on the phone at the same time?

$n = \lambda r = (50000 * 3/(24 * 60)) * 6 = 625$, but simce two people are talking for each job the anser is 1250 persons.

Exercise 3

We would like to model a computer system with a queuing model. We measure the traffic to the system and find that the average arrival rate is A jobs per second. Also, we measure how long time it takes for the system to service each job, and find that the average service time for a job is B seconds.

(a) Assume that there is one CPU in the computer that can process jobs, and that this CPU can be modeled as a server in the queuing model. For what values of A is the system stable?

For $A < \frac{1}{B}$ for most systems. If both the interarrival times and service times are fixed then stability is for $A \leqslant \frac{1}{B}$.

(b) Instead assume that the computer is a multi-processor system, with c CPU:s that each can process the jobs (the CPU:s are modeled as c servers). For what values of A is the system now stable?

For $A < \frac{c}{B}$ for most systems. If both the interarrival times and service times are fixed then stability is for $A \leqslant \frac{c}{B}$.

(c) Assume that the system can be modeled with an infinite queue, and that the stability condition is fulfilled. What is the throughput of the system?

A

(d) What requirements must the arrival process and the service times fulfill if we want to use an M/M/c-model during the analysis?

exponential

(e) We measure the number of jobs in the system, and find that, in average, there are N jobs at the same time, either being served or waiting in queue. Determine the average response time for a job, that is the average time it takes from a job arrives at the system until it leaves the system (assume that the queue is infinite).

$n = \lambda r$, which considering multiple servers gives $r = \frac{N}{cA}$,

Exercise 4

In this exercise we investigate a computer system with one CPU and a job queue. We perform measurements on the system and find that, in average, 10 jobs per second arrive at the system, and that the average response time for a job is 0.09 seconds. The computer processes one job at a time, and the queuing discipline is First-Come-First-Served. First, assume that the job queue is so long that the blocking probability can be assumed to be zero.

(a) Determine the mean number of jobs in this system.

$n = \lambda r$ which gives $r = 10 * 0.9 = 9$ jobs.

(b) Determine the throughput.

10 jobs per second

Exercise 5

A web server has an average service time of 140 ms. The server can hold up to 30 incoming requests in its queue. Assuming exponentially distributed inter-arrival and service times, what is the average arrival rate it can handle while upholding a 0.0001 probability of losing a request?

$Pr\,(K \geq k)$=ρ^k and k=32 gives $\lg(10^{-4}) = k * \lg\rho$ which gives $\rho = 10^{(\frac{log(10-4)}{32}} = 0.74989$ Since $\rho = \frac{\lambda}{\mu}$then $\lambda = \frac{0.74989}{0.140} = 5.356$

Exercise 6

During the Karlstad fair, there was a popular hot-dog stand that sold cheap hot-dogs. You friend Nils, who had a break from his work at manpower, watched the stand for 30 minutes. He told you that in the 30 minute period, there were 23 customers, and that the hot-dog salesman could only rest for 4 minutes. Assume that the customer inter-arrival and the service times are exponentially distributed and IID to the observation.

(a) In average, how many customers per day visited the stand if it was open for 12 hours a day?

$23 * 2 * 12 = 552$

(b) Calculate the average number of customers in the queue.

$n_q = \frac{\rho^2}{1-\rho}$ for M/M/1 $\rho = U = 26/30$ and thus n_q=5.66663.

(c) Calculate the mean response time.

$r = \frac{1}{\mu-\lambda} = \frac{1}{23/26-23/30} = 8.478$ which is the same as $r = \frac{n}{\lambda} = \frac{n_q+n_s}{\lambda} = \frac{n_q+\rho}{\lambda} = \frac{6.5}{23/30}$

(d) Determine the probability that a customer could buy his hot-dog without waiting.

1-$\rho = 1 - U = 1 - (4/30) = 0.133333$

Exercise 7

You have invested in an expensive database server that the manufacturer said would have an average service rate of 5 requests per second. The server has an internal queue of 1000 requests(≈infinite). When you measure the response time using a load generator which generates jobs with exponentially distributed inter-arrival time with a mean of 0.24 seconds you get the results shown below. Do you think made a good investment? What can you say about the service time distribution, and about the claims of the manufacturer?

0.4317 0.8038 1.3132 3.2768 0.7868 0.4388 0.8817 3.4869 1.3343 0.7516 2.1473 1.2286

Assuming that the service times are exponentially distributed (which the widely varying reponse times suggest), the results seem to verify the claims of the manufacturer, although with no statistical significance since the exponential distribution has so large variance. The measured $\bar{r} = 1.4068$. For M/M/1 then $r = \frac{1}{\mu - \lambda}$ which leads to $\mu = \frac{1}{r} + \lambda = \frac{1}{1.4068} + \frac{1}{0.24} = 4.8775$requests/s which is quite close to the claimed value.

Exercise 8, Bonus exercise

Write an octave funktion to calulate values for a M/M/C system. Talk with the instructor for further details on this assignment. (See pg 234-235 in lilja or pg 200 Stallings)