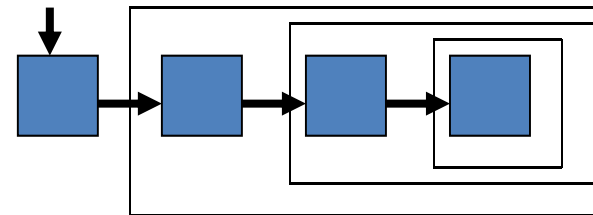


Recursion: Lab 2 examples

- **An entity partially defined** in terms of itself is recursively defined
- **A function which conditionally calls** itself is recursively defined

- Ex: Linked List



- **List ::= Head Tail | α**
Head ::= element
Tail ::= List
- **List is either empty or non-empty**

Recursion: add to a sequence

S ::= **head tail** | **empty**; **head** ::= element; **tail** ::= **S**

```
static listref be_add_val(listref L, int v)
{
    return is_empty(L)          ? create_e(v)
       : v < get_value(head(L)) ? cons(create_e(v), L)
       : cons(head(L), be_add_val(tail(L),v));
}
```

→ reconstructor (cons) & destructors (head & tail)

[Sequence pattern]

1. **Empty** case
2. Non-empty, non-recursive (**head**)
3. Non-empty, recursive (**tail**)

This “forces” a certain abstract programming style

[Recursion: add to a BST]

- Haskell

bAdd v []	= v: []	// empty
bAdd v [x:xs]		
v < x	= v : [x:xs]	// head
otherwise	= x : bAdd v xs	// tail

Recursion: add to a BST

```
static treeref b_add(treeref T, int v)
{
  return is_empty(T)      ? create_node(v)
     : v < get_value(node(T)) ?
           cons(b_add(LC(T), v), node(T), RC(T))
     : v > get_value(node(T)) ?
           cons(LC(T), node(T), b_add(RC(T), v))
     : T;
}
```

re-constructor

cons

de-constructors

LC, node, RC

[BST pattern]

1. **Empty** case
2. Non-empty case, recursive **LC**
3. Non-empty case, recursive **RC**
4. Non-empty case, non recursive **node**

This “forces” a certain abstract programming style

Recursion: remove from a BST

```
static treeref b_rem(treeref T, int v)
{
  return is_empty(T)      ? T
     : v < get_value(node(T)) ?
           cons(b_rem(LC(T), v), node(T), RC(T))
     : v > get_value(node(T)) ?
           cons(LC(T), node(T), b_rem(RC(T), v))
     :
       removeAtRoot(T);
}
```

BT ::= LC node RC | empty; node ::= element; LC, RC ::= BT

Recursion: remove from a BST

```
static treeref removeAtRoot(treeref T) // 00 01 10 11
{
  return is_empty(LC(T)) ? RC(T) // 00 01
    : is_empty(RC(T)) ? LC(T) // 10
    : HDiff(T) > 0 ? LCmaxAsRoot(T) // 11
    : RCminAsRoot(T);
}
```

A slightly different pattern!

NB: LCmaxAsRoot & RCminAsRoot will call **cons & **b_rem**!**

Recursion: remove

- E.g. tree ((1 3 4) 5 (6 7 8)) – remove 5
- **b_rem** → removeAtRoot (2 children) →
 - RCminAsRoot (6)
 - cons((1, 3, 4), 6, **b_rem((6, 7, 8), 6)**)
 - **cons(b_rem((6), 6), 7, 8)**
 - **removeAtRoot LC=α, RC=α**
 - **cons(*, 7, 8)**
 - **cons((1, 3, 4), 6, (*, 7, 8))** --- result

[Abstraction...]

- ... hides the “**mechanics**” of the implementation
- ... provides a “**thinking tool**” for “abstract programming”
- ...thinking in terms of **references** and **values**
- ... requires practise
- ... in the end is a more **efficient method** of programming