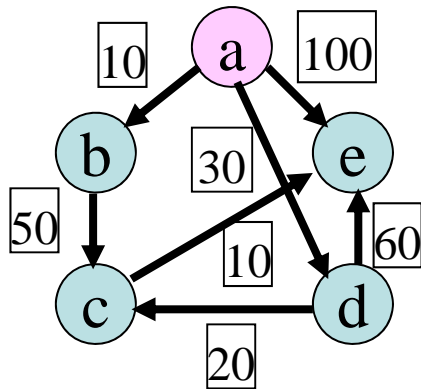


# Dijkstra - overview

## Graph

- 7 edges



## Cost Matrix

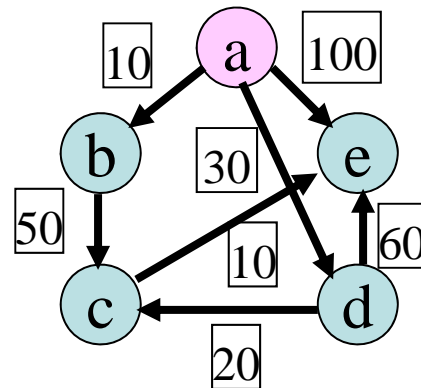
	a	b	c	d	e
a		10	§	30	100
b	§		50	§	§
c	§	§		§	10
d	§	§	20		60
e	§	§	§	§	

# Dijkstra - overview

## Graph

- 7 edges

- a → b 10
- a → d 30
- a → e 100
- b → c 50
- c → e 10
- d → c 20
- d → e 60

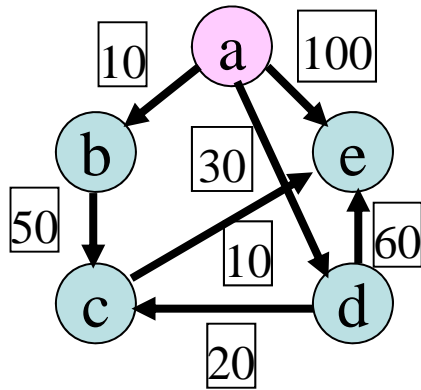


## Paths a to {b,c,d,e}

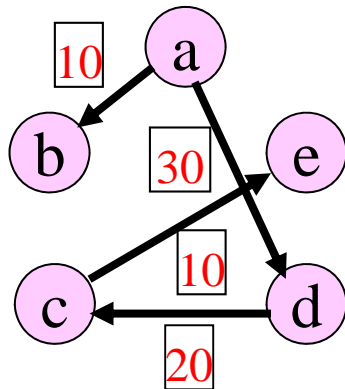
- 8 paths

- a → b 10
- a → b → c 60
- a → d → c 50 (shortest path)
- a → d 30
- a → e 100
- a → b → c → e 70
- a → d → e 90
- a → d → c → e 60 (shortest path)

# Dijkstra - overview



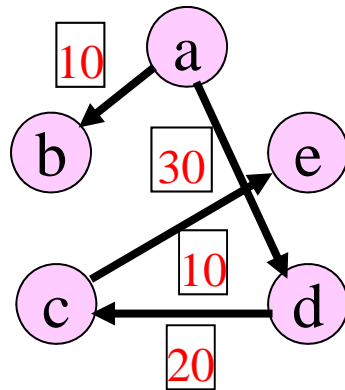
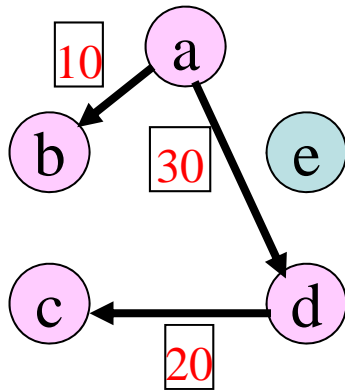
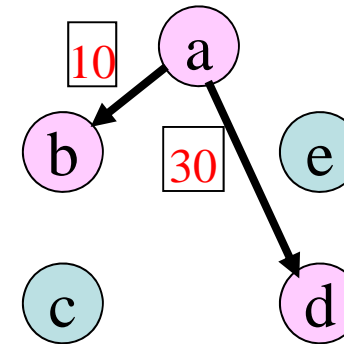
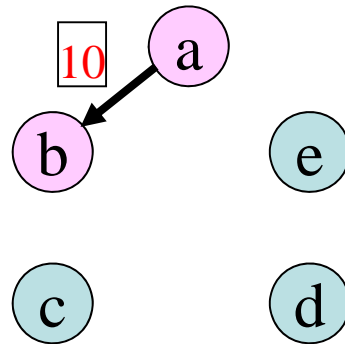
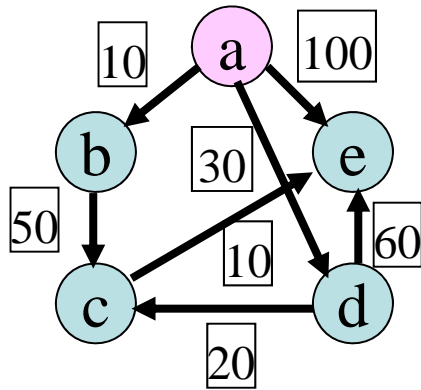
Look at the paths		
a → b	<b>a → b</b> (10)	a → <b>b → c</b> (60)
a → d	<b>a → d</b> (30)	a → <b>d → c</b> (50)   a → <b>d → e</b> (90)
a → c	§ infinite	a → d → <b>c → e</b> (60)
a → e	<b>a → e</b> (100)	



a	<b>b</b>	c	d	e	a	<b>b</b>	c	<b>d</b>	e
i	10	§	30	100	ii	10	<b>60</b>	30	100

a	<b>b</b>	<b>c</b>	<b>d</b>	e	a	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>
iii	10	<b>50</b>	30	<b>90</b>	iv	10	<b>50</b>	30	<b>60</b>

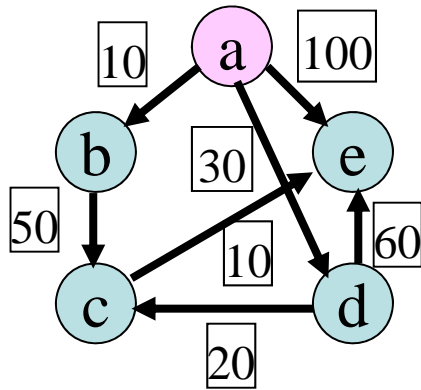
# Dijkstra - overview



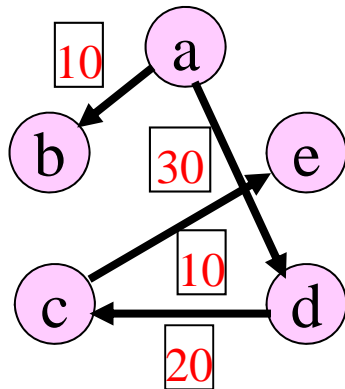
a	b	c	d	e
i	10	∞	30	100

a	b	c	d	e
iv	10	<b>50</b>	30	<b>60</b>

# Dijkstra - overview



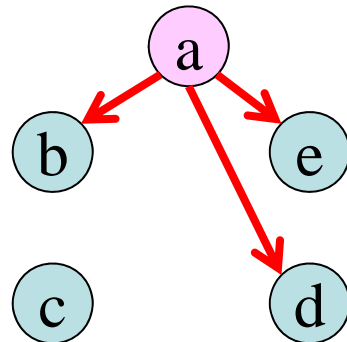
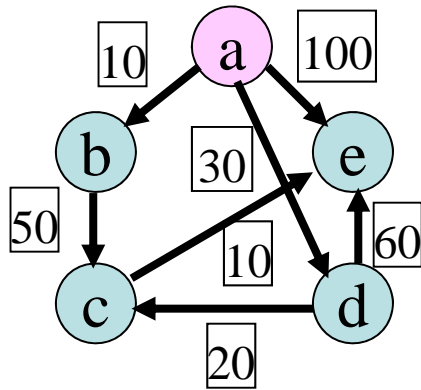
Look at the paths		
a → b	<b>a → b</b> (10)	a → <b>b → c</b> (60)
a → d	<b>a → d</b> (30)	a → <b>d → c</b> (50)   a → <b>d → e</b> (90)
a → c	§ infinite	a → d → <b>c → e</b> (60)
a → e	<b>a → e</b> (100)	



a	b	c	d	e
	10	<b>50</b>	30	<b>60</b>
a	<b>d</b>	a	<b>c</b>	
10	<b>20</b>	30	<b>10</b>	

SPT Shortest Path Tree
← Distance from a
← Edges
← Edge costs

# Initial State

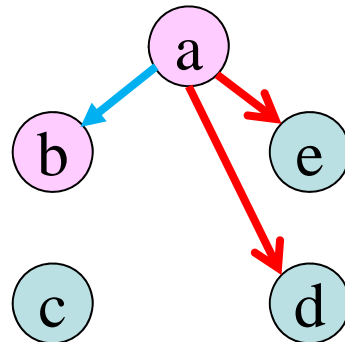
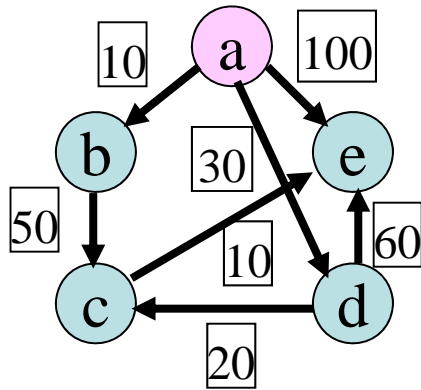


$V = \{a, b, c, d, e\}$   
 $S = \{a\}$   
 $V-S = \{b, c, d, e\}$

	<b>a</b>	b	c	d	e
D (distance)		10	§	30	100

D represents the path length  
 § = infinity i.e. no edge

# Iteration 1: initialisation



$V = \{a, b, c, d, e\}$   
 $S = \{a, b\}$   
 $V-S = \{c, d, e\}$

	a	b	c	d	e
D (distance)		10	∞	30	100

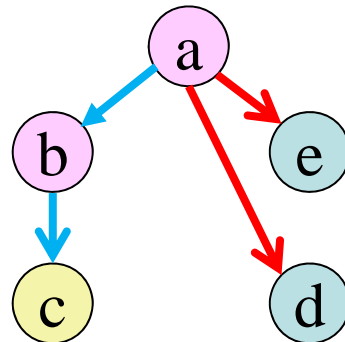
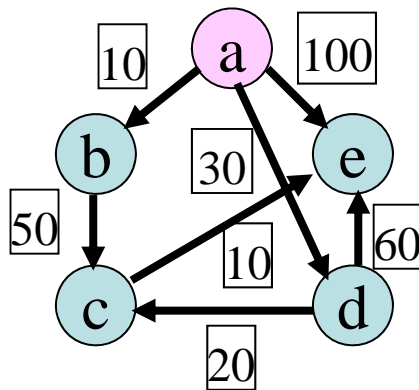
Choose minimum  $\{b, c, d, e\} \rightarrow b$

**Cheapest path from node a**

**(here edge  $a \rightarrow b$ )**

**(a, b, 10)**

# Iteration 1: recalculate path costs **via b**



$V = \{a, b, c, d, e\}$   
 $S = \{a, b\}$   
 $V-S = \{c, d, e\}$

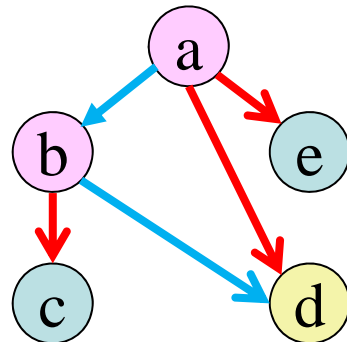
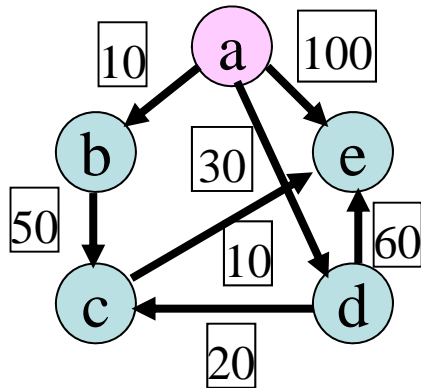
	a	b	c	d	e
D (distance)		10	<b>60</b>	30	100

Path from  
**[a → b → c]**

$$D[c] = \min(D[c], D[b] + C[b,c]) \rightarrow \min(\text{\textcircled{S}}, 10+50) \rightarrow 60 \text{ --- change}$$



# Iteration 1: recalculate path costs **via b**



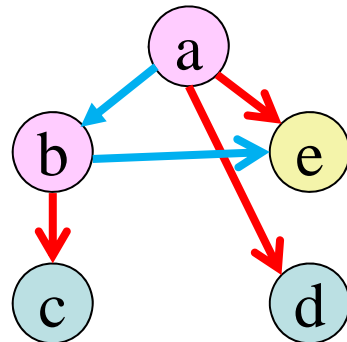
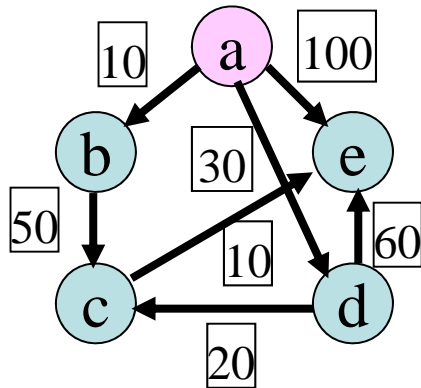
$V = \{a, b, c, d, e\}$   
 $S = \{a, b\}$   
 $V-S = \{c, d, e\}$

	a	b	c	d	e
D (distance)		10	60	30	100

Path from  
**[a → b → d]**

$$D[d] = \min(D[d], D[b] + C[b,d]) \rightarrow \min(30, 10 + \text{\$}) \rightarrow 30 \quad \text{--- no change}$$

# Iteration 1: recalculate path costs **via b**



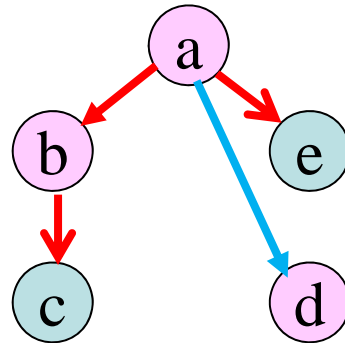
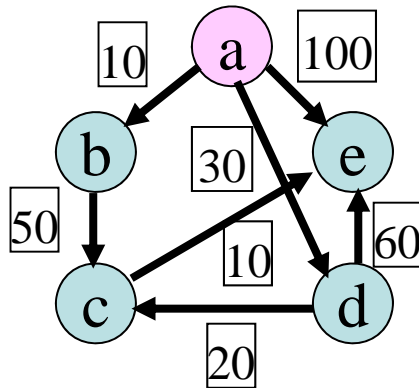
$V = \{a, b, c, d, e\}$   
 $S = \{a, b\}$   
 $V-S = \{c, d, e\}$

	a	b	c	d	e
D (distance)		10	60	30	100

Path from  
**[a→b→e]**

$$D[e] = \min(D[e], D[b] + C[b,e]) \rightarrow \min(100, 10+\text{\textcircled{3}}) \rightarrow 100 \quad \text{--- no change}$$

# Iteration 2: initialisation



$V = \{a, b, c, d, e\}$

$S = \{a, b, d\}$

$V-S = \{c, e\}$

	a	b	c	d	e
D (distance)		10	60	30	100

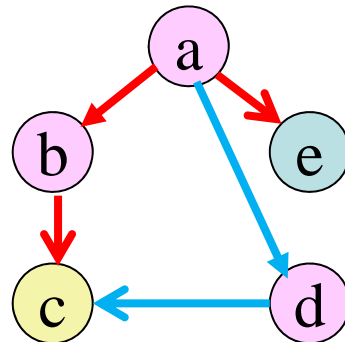
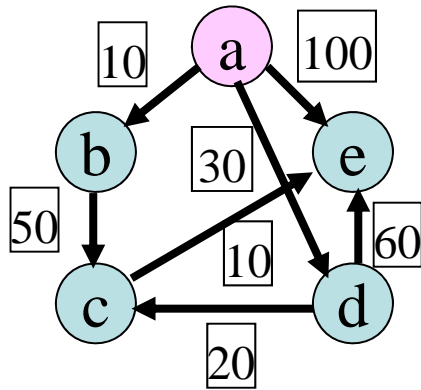
Choose minimum  $\{c, d, e\} \rightarrow d$

**Cheapest path from node [a]**

**(here edge  $a \rightarrow d$ )**

**(a, d, 30)**

# Iteration 2: recalculate path costs **via d**



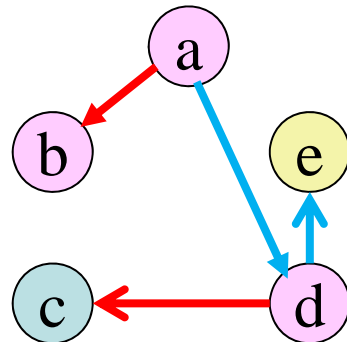
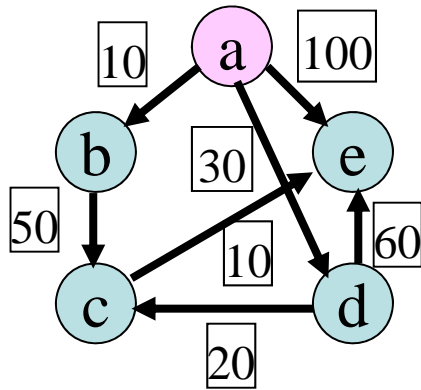
$V = \{a, b, c, d, e\}$   
 $S = \{a, b, d\}$   
 $V-S = \{c, e\}$

	a	b	c	d	e
D (distance)		10	50	30	100

Path from [a→d→c]

$$D[c] = \min(D[c], D[d] + C[d,c]) \rightarrow \min(60, 30+20) \rightarrow 50 \text{ --- change}$$

# Iteration 2: recalculate path costs **via d**



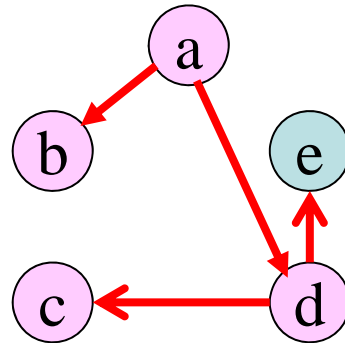
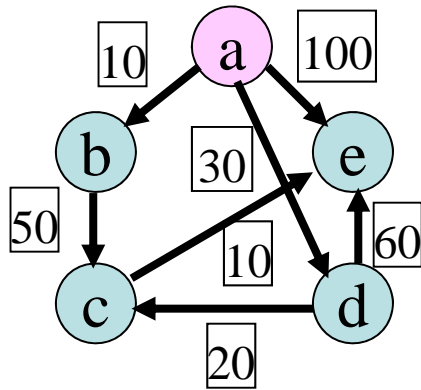
$V = \{a, b, c, d, e\}$   
 $S = \{a, b, d\}$   
 $V-S = \{c, e\}$

	a	b	c	d	e
D (distance)		10	50	30	90

Path from [a→d→e]

$$D[e] = \min(D[e], D[d] + C[d,e]) \rightarrow \min(100, 30+60) \rightarrow 90 \text{ --- change}$$

# Iteration 3: initialisation



$V = \{a, b, c, d, e\}$

$S = \{a, b, d, c\}$

$V-S = \{e\}$

	a	b	c	d	e
D (distance)		10	50	30	90

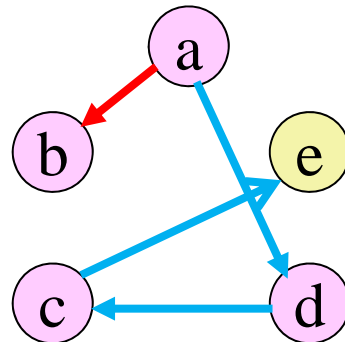
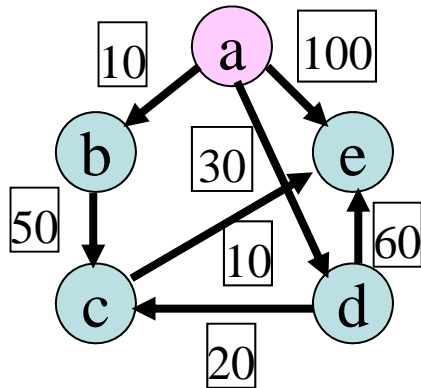
Choose minimum  $\{c, e\} \rightarrow c$

**Cheapest path from node [a]**

**(here path  $a \rightarrow d \rightarrow c$ )**

**$(a, d, 30) + (d, c, 20)$**

# Iteration 3: recalculate path costs **via c**



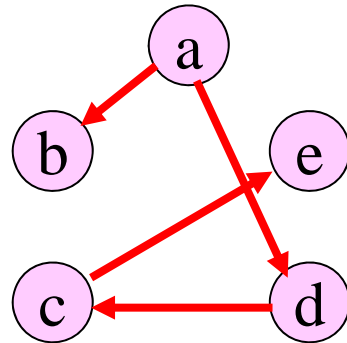
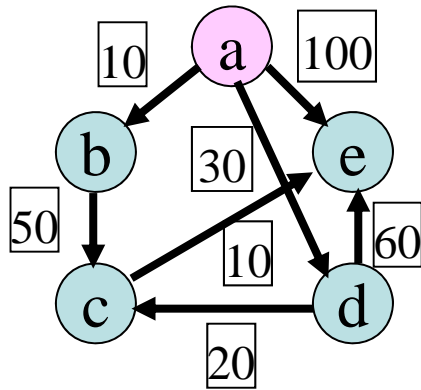
$V = \{a, b, c, d, e\}$   
 $S = \{a, b, d, c\}$   
 $V-S = \{e\}$

	a	b	c	d	e
D (distance)		10	50	30	60

**Cheapest path from a → d → c to e**

$$D[e] = \min(D[e], D[c] + C[c,e]) \rightarrow \min(90, 50+10) \rightarrow 60 \text{ --- change}$$

# Iteration 4: initialisation/final result



$V = \{a, b, c, d, e\}$

$S = \{a, b, d, c, e\}$

$V-S = \{ \}$

	a	b	c	d	e
D (distance)		10	50	30	60

Choose minimum  $\{e\} \rightarrow e$

**Cheapest path from node [a]**

**$a \rightarrow d \rightarrow c \rightarrow e$**

**V-S is empty**